

Functional medium-dependence of the nonrelativistic optical model potential

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Abstract

By examining the structure in momentum and coordinate space of a two-body interaction spherically symmetric in its local coordinate, we demonstrate that it can be disentangled into two distinctive contributions. One of them is a medium-independent and momentum-conserving term, whereas the other is functionally –and exclusively– proportional to the radial derivative of the reduced matrix element. As example, this exact result was applied to the unabridged optical potential in momentum space, leading to an explicit separation between the medium-free and medium-dependent contributions. The latter does not depend on the strength of the reduced effective interaction but only on its variations with respect to the density. The modulation of radial derivatives of the density enhances the effect in the surface and suppresses it in the saturated volume. The generality of this result may prove to be useful for the study of surface-sensitive phenomena.

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I. INTRODUCTION

During the past two decades, several developments in theoretical nuclear research have allowed significant improvements in the microscopic description of nuclear collisions [1, 2]. Such is the case of nucleon scattering, where continuous efforts have led to detailed microscopic realizations of optical model potentials, thus providing the most complete current first-order nonrelativistic description of nuclear collisions off nuclei over a wide energy range and various targets. These calculations emphasize a detailed treatment of the NN effective interaction, particularly its density dependence as well as its energy and momentum dependence. Quite generally, however, all realizations of the optical potential become a single expression in the form of a convolution of medium-dependent effective interactions folded with the target ground-state mixed density. In this article we demonstrate that the optical potential can be expressed as the sum of two very distinctive terms, one of them depends exclusively on the free-space t matrix and the other as a gradient of the reduced g matrix. This result implies that intrinsic medium effects in optical potentials become enhanced in the nuclear surface and suppressed in the saturated volume.

Since the early realizations of microscopic optical potentials [3–5], the role of nuclear medium effects has been a major issue in the study of the dynamics involved in nucleon-nucleus collisions. In these studies density-dependent nucleon-nucleon local (NN) effective interactions have been developed to represent the force between nucleons in the nuclear medium. The use of these local forces, with suitable local density prescriptions, have led to folding optical potentials in coordinate space which provide reasonable descriptions of NA scattering data at energies between a few tens of MeV up to near 400 MeV. Recent developments [2, 6], within the same philosophy, have succeeded in including non localities in the optical potential stemming from the exact inclusion of the exchange term. In this case the full mixed density from nuclear shell models are used and provide reasonable account of the existing NA scattering data.

A slightly different strategy has been followed by Arellano, Brieva and Love (ABL), with the realization of folding optical potentials in momentum space [7, 8]. In their approach genuine nonlocal g matrices, based on the Brueckner-Bethe-Goldstone infinite nuclear matter model, are folded to the ground-state local density of the target. As a result, nonlocal potentials are obtained with varying degree of success in describing the low and intermediate

energy data.

The inclusion of medium effects has also been addressed within the spectator expansion [9], where the coupling between struck nucleons and target spectators is taken into account. This approach is an extension of the Watson, and Kerman, McManus and Thaler theories, with focus is on the many-body propagator involved in the (A+1)-body problem.

From a more general prospective, various formal expressions of the optical potential can be found in the literature [10–14]. Although they may differ in the way they establish contact with the bare NN potential, they all become a folding expression between the target ground state and a generalized two-body effective interaction. In this article we analyze this general expression and demonstrate that, regardless of the model of utilized to represent the effective interaction, the intrinsic medium effects become manifest in the nuclear surface. The implications of this result are examined in the framework of an infinite nuclear matter model for the NN effective interaction.

This article is organized as follows. In Section II we outline the general framework, discuss the structure of two-body operators and introduce the ‘asymptotic separation’ for spherically symmetric systems. The result is then applied to the unabridged optical potential in momentum space. In Section III we make use of an infinite nuclear matter model for the effective NN interaction and examine its implications in the optical potential. Furthermore, we analyze its consistency with the ABL approach, and assess the medium sensitivity of selected matrix elements at various energies. In Section IV we present a summary and the main conclusions of this work. Additionally, we have added three appendices where we include some intermediate steps.

II. THE OPTICAL POTENTIAL

A general representation of the optical model potential for collisions of a hadronic probe with kinetic energy E off a composite target is given by the expression

$$U(\mathbf{k}', \mathbf{k}) = \int d\mathbf{p}' d\mathbf{p} \langle \mathbf{k}' \mathbf{p}' | \hat{T} | \mathbf{k} \mathbf{p} \rangle \hat{\rho}(\mathbf{p}', \mathbf{p}) , \quad (1)$$

where \hat{T} represents a two-body effective interaction containing, in general, information about the discrete spectrum of the many-body system. The one-body mixed density $\hat{\rho}(\mathbf{p}', \mathbf{p})$ represents the ground-state structure of the target. Thus, a fully consistent evaluation of the

optical potential by means of the full \hat{T} matrix would require the solution of the $(A+1)$ -body system, a formidable task. This difficulty is circumvented by treating separately the ground state and the two-body effective interaction. This separation becomes suitable at intermediate and high energies, where the discrete spectrum of the many-body Green's function is distant from the projectile energy in the continuum. Then, the target ground-state may be described resorting to alternative framework such as Hartree-Fock-Bogoliubov. The effective interaction, in turn, can be modeled using the Brueckner-Bethe-Goldstone approach.

A. Two-body effective interaction

Let us first focus our discussion on the two-body effective interaction and examine its structure. Quite generally, the representation of the two-body operator \hat{T} in either momentum or coordinate space requires the specification of four vectors. We denote the coordinate representation of \hat{T} in the form

$$\langle \mathbf{r}' \mathbf{s}' | \hat{T} | \mathbf{r} \mathbf{s} \rangle = T(\mathbf{r}' \mathbf{s}'; \mathbf{r} \mathbf{s}) ,$$

where the ‘prior’ coordinates of each particle are \mathbf{r} and \mathbf{s} , respectively. Similarly, \mathbf{r}' and \mathbf{s}' refer to the ‘post’ coordinates of the same particles, as shown in Fig. (1a). An alternative set of coordinates is summarized by the transformation

$$\mathbf{R}' = (\mathbf{r}' + \mathbf{s}')/2 ; \quad \mathbf{x}' = \mathbf{r}' - \mathbf{s}' ; \tag{2}$$

$$\mathbf{R} = (\mathbf{r} + \mathbf{s})/2 ; \quad \mathbf{x} = \mathbf{r} - \mathbf{s} ;$$

where \mathbf{x} represents the prior relative coordinate of the pair and \mathbf{R} their respective center of mass, as illustrated in Fig. (1b). With this transformation we express the equivalence $T(\mathbf{r}' \mathbf{s}'; \mathbf{r} \mathbf{s}) = T_{\mathbf{R}' \mathbf{R}}(\mathbf{x}', \mathbf{x})$. Thus, following the procedure outlined in Appendix A we express the momentum space elements $\tilde{T} \equiv \langle \mathbf{k}' \mathbf{p}' | \hat{T} | \mathbf{k} \mathbf{p} \rangle$, in the form

$$\tilde{T} = \int \frac{d\mathbf{Z}}{(2\pi)^3} e^{i\mathbf{Z} \cdot (\mathbf{W}' - \mathbf{W})} g_{\mathbf{Z}}[\tfrac{1}{2}(\mathbf{W}' + \mathbf{W}); \mathbf{b}', \mathbf{b}] , \tag{3}$$

where $g_{\mathbf{Z}}$ represents the reduced interaction at the local coordinate \mathbf{Z} . Here we have denoted

$$\mathbf{W} = \mathbf{k} + \mathbf{p} , \quad \mathbf{b} = \tfrac{1}{2}(\mathbf{k} - \mathbf{p}) ,$$

the prior total and relative momenta, respectively. The same construction applies to the post momenta, where prime marks are used. The relationship between these momenta is illustrated in Fig. (2). Eq. (3) for \tilde{T} also expresses the role of vector \mathbf{Z} ,

$$\mathbf{Z} = \frac{1}{4}(\mathbf{r}' + \mathbf{r} + \mathbf{s}' + \mathbf{s}) .$$

the center of gravity of the four coordinates of the two particles. We name this the *local coordinate*, the *locus* where the reduced interaction is evaluated. Notice that this coordinate is invariant under the permutation of coordinates $\mathbf{r} \rightleftharpoons \mathbf{s}$.

The above representation of the \hat{T} matrix displays very clearly its dependence in terms of the total (\mathbf{W} , \mathbf{W}') and relative (\mathbf{b} , \mathbf{b}') momenta. Additionally, the Wigner transform in the \mathbf{R} and \mathbf{R}' coordinates restricts further the structure of T , suggesting the definitions

$$\mathbf{W}_\perp \equiv \mathbf{W}' - \mathbf{W} = (\mathbf{p}' - \mathbf{p}) - (\mathbf{k} - \mathbf{k}') ; \quad (4a)$$

$$\mathbf{W}_\parallel \equiv \frac{\mathbf{W}' + \mathbf{W}}{2} = \frac{1}{2}(\mathbf{k}' + \mathbf{k} + \mathbf{p}' + \mathbf{p}) . \quad (4b)$$

The vector \mathbf{W}_\perp represents the total momentum gained by the pair upon interaction ($\mathbf{W}' = \mathbf{W} + \mathbf{W}_\perp$), whereas \mathbf{W}_\parallel is the average of the prior and post total momenta. These momenta become perpendicular only if $\mathbf{W}'^2 = \mathbf{W}^2$.

B. Asymptotic separation of \tilde{T}

We now examine the structure of \tilde{T} in the context of a finite nucleus with spherical symmetry. By that we understand that g_Z depends only on the magnitude of the local coordinate, $|\mathbf{Z}| = Z$. Additionally, let us assume that as $Z \rightarrow \infty$, g_Z tends to its free-space form g_∞ . If we omit the three vector arguments of g_Z and decompose it as $g_Z = (g_Z - g_\infty) + g_\infty$, then

$$\tilde{T} = \delta(\mathbf{W}_\perp)g_\infty + \frac{1}{(2\pi)^3} \int d\mathbf{Z} e^{i\mathbf{Z} \cdot \mathbf{W}_\perp} (g_Z - g_\infty) .$$

Carrying out the solid angle integration, the integrand of the remaining radial integral is simply $4\pi Z^2 j_0(ZW_\perp)(g_Z - g_\infty)$, which integrated by parts yields the asymptotic separation

$$\tilde{T} = \delta(\mathbf{W}_\perp)g_\infty - \frac{1}{2\pi^2} \int_0^\infty Z^3 dZ \Phi_1(ZW_\perp) \frac{\partial g_Z}{\partial Z} . \quad (5)$$

Here δ denotes the Dirac delta function and Φ_1 represents the profile function defined by

$$\Phi_1(t) = j_1(t)/t ,$$

with j_1 the spherical Bessel function of order 1. In Fig. (3) we plot Φ_1 , where we observe that its peak value (1/3) occurs at the origin, and that it is mainly contained within the region $t \lesssim 4$.

What is interesting about Eq. (5) is that it separates unambiguously the free-space contribution of the \hat{T} matrix from its medium-dependent counterpart. On the one hand, the medium dependence enters solely as the gradient of the reduced element while the total momentum is not conserved. On the other hand, the medium-independent contribution does conserve momentum, as dictated by $\delta(\mathbf{W}_\perp)$. This contrast is physically consistent with our notion about non-translational invariant systems. By introducing a Z -dependent reduced interaction, the two-body \hat{T} matrix does not express conservation of the total momentum. The conservation becomes manifest only if $\partial g_Z / \partial Z = 0$, as in the cases of interacting nucleons in infinite nuclear matter or free space. More interestingly, the result displayed by Eq. (5) is sufficiently general to allow us to model the medium dependence in a finite nucleus and justifies the incorporation of medium effects in distorted wave Born approximations (DWBA).

The application of the asymptotic separation in Eq. (1) for U yields some undisclosed features. Let us first change variables from \mathbf{p}, \mathbf{p}' to \mathbf{P}, \mathbf{Q} ,

$$\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2 ; \quad \mathbf{Q} = \mathbf{p}' - \mathbf{p} ;$$

so that $d\mathbf{p}' d\mathbf{p} = d\mathbf{Q} d\mathbf{P}$. These two vectors represent the mean and transferred struck-nucleon momenta, and the integration on them accounts for the Fermi motion of the target nucleons. Analogously, let us denote

$$\mathbf{K} = (\mathbf{k} + \mathbf{k}')/2 ; \quad \mathbf{q} = \mathbf{k} - \mathbf{k}' ;$$

so that $\mathbf{W}_\perp = \mathbf{Q} - \mathbf{q}$. With this notation we re-express the vector arguments of the reduced g matrix,

$$g_Z(\mathbf{K}_\parallel, \mathbf{b}', \mathbf{b}) \rightarrow g_Z(\mathbf{K} + \mathbf{P}, \boldsymbol{\kappa}_-, \boldsymbol{\kappa}_+) ,$$

where

$$\boldsymbol{\kappa}_\pm = \frac{1}{2}[\mathbf{K} - \mathbf{P} \pm \frac{1}{2}(\mathbf{q} + \mathbf{Q})] . \quad (6)$$

With these considerations and using Eq. (5) for the two-body interaction, the *unabridged* [22] optical potential takes the form

$$U = U_0 + U_1 , \quad (7)$$

with

$$U_0 = \int d\mathbf{P} \hat{\rho}(\mathbf{q}; \mathbf{P}) g_\infty , \quad (8a)$$

$$U_1 = -\frac{1}{2\pi^2} \int d\mathbf{Q} d\mathbf{P} \hat{\rho}(\mathbf{Q}; \mathbf{P}) \times \int_0^\infty Z^3 dZ \Phi_1(Z|\mathbf{Q} - \mathbf{q}|) \frac{\partial g_Z}{\partial Z} . \quad (8b)$$

The first term, U_0 , depends exclusively on the medium-free reduced matrix, whereas the second depends on the gradient of g .

III. NUCLEAR MATTER MODEL

In the preceding analysis we have made no mention to a specific approach to model the \hat{T} matrix. In this regard, the framework is general enough to include various strategies to describe an effective two-body interaction in the realm of a finite nucleus. However, if the reduced matrix g is taken [7] as the antisymmetrized Brueckner-Bethe-Goldstone reaction matrix of starting energy E ,

$$g(E) = v + v \frac{\hat{Q}}{E + i\eta - \hat{h}_1 - \hat{h}_2} g(E) ,$$

then the reduced matrix at infinity, g_∞ , corresponds to the free scattering matrix $t(E)$. In the above equation \hat{h}_1 and \hat{h}_2 correspond to quasi-particle energies and \hat{Q} , the Pauli blocking operator. Therefore, the first term of the optical potential in Eq. (7) becomes

$$U_0(E) = \int d\mathbf{P} \hat{\rho}(\mathbf{q}; \mathbf{P}) t(E) ,$$

the lowest-order free t matrix full-folding optical potential in the Watson and Kerman-McManus-Thaler approach [12, 15]. Actual calculations of this contribution were realized in the early nineties [16–18].

With regard to U_1 we note that the g matrix in Brueckner-Bethe-Goldstone approach is a functional of the density, $g = g[\rho]$. If the reduced matrix is evaluated at a density ρ specified by the local coordinate \mathbf{Z} , then

$$\frac{\partial g_Z}{\partial Z} = \left(\frac{\partial \rho}{\partial Z} \right) \times \frac{\partial g}{\partial \rho} . \quad (9)$$

Considering that $\rho'(Z) \equiv \partial \rho / \partial Z$ peaks in the nuclear surface, the intrinsic medium-dependent contributions to the optical potential become accentuated in that region. The

strength of such contributions will depend on $\partial g(E)/\partial \rho$, an energy-dependent operator in spin-isospin space.

To focus these ideas, in Fig. (4) we characterize the proton and neutron densities in ^{208}Pb [19], where in the upper frame we plot the proton and neutron density, in the middle frame the local Fermi momentum given by

$$k_F = (3\pi^2\rho)^{1/3} ,$$

and in the lower frame the negative gradient of the density. We have multiplied this function by Z^3 to account for its actual weight in the radial integration.

What becomes clear from this figure is that medium effects stemming from $\partial g/\partial \rho$ become dominant (if non zero) in the region between 5.5 fm and 9 fm. In this range the local Fermi momentum k_F varies between 0.3 fm^{-1} and 1.2 fm^{-1} , suggesting the densities at which the g matrix needs to be evaluated.

A. Contact with the ABL approach

The ABL approach to the optical potential constitutes an extension of the early full-folding approach based on the free t matrix [7]. This extension makes use of an infinite nuclear-matter model to represent the NN effective interaction between the projectile and the target nucleons. Upon the use of a simplifying assumption regarding its relative momenta dependence, and resorting to the Slater approximation of the mixed density, the optical potential takes the form of a folding of the diagonal (local) density with a nonlocal Fermi-averaged g matrix. We verify this result as a limit case of Eq. (8b) for the unabridged optical potential.

If we assume that the g matrix in Eq. (8b) exhibits a weak dependence on the transferred momentum \mathbf{Q} , then the relative momenta in g can be evaluated at $\mathbf{Q} = \mathbf{q}$, consistent with the peak of Φ_1 at the origin. With these considerations we set

$$\kappa_{\pm} \rightarrow \kappa_{\pm}^{(0)} \equiv \frac{1}{2}(\mathbf{K} - \mathbf{P} \pm \mathbf{q}) , \quad (10)$$

and symbolize

$$g_Z(E) \rightarrow g_Z^{(0)}(E)$$

Additionally, if we use (B1) for the Slater approximation to $\hat{\rho}(\mathbf{Q}; \mathbf{P})$, then

$$\begin{aligned}
U_1 = & -\frac{2}{\pi} \int_0^\infty Z^3 dZ \int_0^\infty Z'^2 dZ' \rho(Z') \times \\
& \int d\mathbf{Q} \Phi_1(Z|\mathbf{Q} - \mathbf{q}|) j_0(QZ') \times \\
& \int d\mathbf{P} \frac{\partial g_Z^{(0)}(E)}{\partial Z} S_F(P; Z').
\end{aligned} \tag{11}$$

Here the rightmost integral does not depend on \mathbf{Q} , therefore the $d\mathbf{Q}$ integration involving $\Phi_1 j_0(QZ')$ can be performed separately. Using Eq. (C2) in Appendix C and reordering the integrals we obtain

$$\begin{aligned}
U_1 = & -4\pi \int_0^\infty Z'^2 dZ' j_0(qZ') \rho(Z') \int d\mathbf{P} S_F(P; Z') \times \\
& \int_0^\infty dZ \Theta(Z - Z') \frac{\partial g_Z^{(0)}(E)}{\partial Z}.
\end{aligned} \tag{12}$$

The integration over Z is immediate. If we identify $g_\infty^{(0)}(E) = t(E)$, the free t matrix, then

$$\begin{aligned}
U_1 = & -4\pi \int_0^\infty Z'^2 dZ' j_0(qZ') \rho(Z') \times \\
& \int d\mathbf{P} S_F(P; Z') [t(E) - g_{Z'}^{(0)}(E)].
\end{aligned} \tag{13}$$

The integral involving the free t matrix leads to $U_0(E)$, whereas the one involving $g_{Z'}^{(0)}(E)$ leads to the ABL *in-medium* folding potential $U_{ABL}(E)$, $U_1 \rightarrow -U_0 + U_{ABL}$, with

$$U_{ABL}(E) = 4\pi \int_0^\infty Z'^2 dZ' j_0(qZ') \rho(Z') g_{Z'}^{(0)}(E). \tag{14}$$

When this term is substituted in Eq. (7) we obtain $U(E) \rightarrow U_{ABL}(E)$, the expected limit. As seen, this result illustrates the fact that U_{ABL} can be rigorously derived from Eq. (8b) using a few simplifying assumptions, i.e. Slater approximation of the mixed density and a weak dependence of the g matrix in the \mathbf{Q} momentum.

B. Contrast with the $\rho(\partial/\partial\rho)$ rearrangement term

During the course of this work it was noticed by some colleagues certain resemblance between Eq. (8b) for the unabridged optical potential, and medium corrections in the form of a $\rho(\partial/\partial\rho)$ term proposed by Cheon and collaborators [20]. It becomes appropriate, therefore, to point out the differences in context and form of this apparent resemblance.

The $\rho(\partial/\partial\rho)$ term of Cheon *et al.* emerges after a perturbative treatment of the transition density for inelastic scattering. In this case the optical potential for elastic scattering is given schematically by $U_{opt} = G(\rho)\rho$, with $G(\rho)$ the Brueckner G matrix and ρ the ground state mixed density. The transition potential for inelastic scattering, U_{tr} , is expressed as $U_{tr} = (\partial U_{opt}/\partial\rho)\rho_{tr}$, with ρ_{tr} the transition density. Combining these equations it is shown that the transition potential can be expressed as

$$U_{tr} = \left[G(\rho) + \rho \frac{\partial G(\rho)}{\partial \rho} \right] \rho_{tr} .$$

As observed, the optical potential U_{tr} results the sum of the elastic term with a corrective term of the form $\rho(\partial/\partial\rho)$. This correction accounts for the rearrangement of the target nucleons in an inelastic process. Additionally, the $\rho(\partial/\partial\rho)$ form of the correction implies relatively uniform contributions in the nuclear interior, in contrast with the $\rho'(\partial/\partial\rho)$ term in Eq. (8b) for *elastic scattering*, where the intrinsic medium effects manifest dominantly in the nuclear surface. The unabridged optical potential discussed here represents elastic processes and its extension to inelastic scattering would require further analysis.

C. Medium sensitivity

The actual evaluation of the unabridged optical potential [c.f. Eq. (8b)] constitutes a very challenging task beyond the scope of this work. Indeed, each matrix element requires the realization of a 7-dimensional integration, three more dimensions than current calculations in the ABL approach. However, it is possible to assess the relative importance of selected terms in the medium-dependent U_1 contribution. In order to isolate the role of medium effects, let us define the amplitude

$$\Gamma(Z'; E, Z) \equiv \int d\mathbf{P} S_F(P; Z') \frac{\partial g_Z^{(0)}(E)}{\partial Z} . \quad (15)$$

In the context of Eq. (12) this amplitude accounts for the Fermi average of the gradient of the effective interaction in the limit $\kappa_{\pm} \rightarrow \kappa_{\pm}^{(0)}$. Thus, it is reasonable to expect that this quantity accounts for the leading contributions stemming from the $d\mathbf{Q}$ integral in Eq. (8b).

In addition to Z' , Z and E , the Γ amplitude depends on the momenta \mathbf{k} and \mathbf{k}' , and spin-isospin degrees of freedom. To examine its radial behavior, we have evaluated the diagonal ($Z = Z'$) elements which are plotted in Fig. (5) for the central pp (upper frames) and pn

(lower frames) channels. The real and imaginary components are shown in the left and right panels, respectively. These amplitudes are evaluated on-shell ($E = k^2/2m = k'^2/2m$), at forward angles, for nucleon energies of 65 MeV (solid curves), 100 MeV (long-dashed curves), 200 MeV (short-dashed curves) and 300 MeV (dotted curves). The density considered in this analysis is that of ^{208}Pb , as in Fig. (4).

By simple inspection we observe that medium effects accounted for by U_1 add attraction and absorption to the medium-free U_0 contribution. All of them are localized in the nuclear surface, becoming weaker in the interior. Additionally, the various components of Γ exhibit distinctive features regarding their energy and medium sensitivity. The energy dependence is identified by the separation of among all four curves, being this most pronounced in the case of $\text{Re}\Gamma_{pn}(E)$ and followed by $\text{Re}\Gamma_{pp}(E)$. The weakest energy dependence occurs for the absorptive component in the pn channel.

Regarding the intrinsic medium effects, the strongest dependence occurs for the real pn amplitude at 65 MeV. In contrast, the weakest dependence occurs for the absorptive pp (upper-right frame) amplitude. Additionally, with the exemption of $\text{Im}\Gamma_{pn}(E)$, all the other amplitudes exhibit decreasing strength with increasing energy.

IV. SUMMARY AND CONCLUSIONS

We have examined the role of medium effects in the optical model potential for hadron-nucleus scattering. The analysis is based on a close scrutiny of the structure of the two-body effective interaction with spherical symmetry in its local coordinate \mathbf{Z} , leading to its asymptotic separation. As a result, we demonstrate that the unabridged optical potential can be separated into two very distinctive contributions. One of them is a momentum-conserving and medium-independent term, while the other is functionally proportional to the radial derivative of the reduced matrix element. If the Brueckner-Hartree-Fock g matrix is used to model the NN effective interaction, then the medium-independent term of the optical potential corresponds to the well known Watson-KMT lowest order full-folding optical model potential, while the medium-dependent term depends exclusively on the gradient of the reduced g matrix. The modulation by the radial derivatives of the density enhances that effect in the nuclear surface and suppresses it in the saturated volume.

The assessment of the intrinsic medium effects by means of the $\Gamma(Z'; E, Z)$ amplitude

points to stronger medium sensitivity in the real part of the pn amplitude at 65 MeV, in contrast with the absorptive pp component. The energy-dependence of these effects are relatively weak in the absorptive pp and pn channels. The introduction of the Γ amplitude may prove to be very useful for the assessment of intrinsic medium effects, such as those associated with the inclusion of non spherical components of the Pauli blocking studied in Ref. [21].

Although we have not provided a full realization of the unabridged optical potential, we have been able to extract its exact functional dependence in terms of the nuclear density when the system is spherically symmetric. This feature may become useful in the context of semi-phenomenological approaches, where the free-space contribution may well be represented in terms of impulse-approximation-like potentials, while the medium dependent term can be modeled as function of $\rho'\partial/\partial\rho$ couplings. At a more fundamental level, it would be interesting to identify missing features in the ABL approach relative to the unabridged potential. As inferred from Fig. (5), intrinsic medium effects become most pronounced –in the nuclear surface– at lower energies. This surface-sensitive phenomenon may be of particular importance in the study of rare isotope beams, where highly unstable nuclei are collided against hydrogen targets. In this case, the traditional intermediate energy regime is reached with 30A-100A MeV beams.

APPENDIX A: REPRESENTATION OF QUANTUM TWO-BODY OPERATORS

Let A a two-body operator and denote its coordinate representation by $\langle \mathbf{r}'\mathbf{s}' | A | \mathbf{r} \mathbf{s} \rangle = A(\mathbf{r}'\mathbf{s}'; \mathbf{r} \mathbf{s})$, where \mathbf{r}, \mathbf{s} (\mathbf{r}', \mathbf{s}') represent the prior (post) coordinates of each particle. An alternative set of coordinates is summarized by Eqs. (2) and illustrated in Fig. (1b). With this transformation we can express $A(\mathbf{r}'\mathbf{s}'; \mathbf{r} \mathbf{s}) = A_{\mathbf{R}'\mathbf{R}}(\mathbf{x}', \mathbf{x})$. If we denote $\tilde{A} \equiv \langle \mathbf{k}'\mathbf{p}' | A | \mathbf{k} \mathbf{p} \rangle$, then

$$\tilde{A} = \frac{1}{(2\pi)^6} \int d\mathbf{r}' d\mathbf{s}' d\mathbf{r} d\mathbf{s} \times e^{-i(\mathbf{k}'\cdot\mathbf{r}'+\mathbf{p}'\cdot\mathbf{s}'-\mathbf{k}\cdot\mathbf{r}-\mathbf{p}\cdot\mathbf{s})} A(\mathbf{r}'\mathbf{s}'; \mathbf{r} \mathbf{s}) .$$

In terms of the coordinate set defined in Eqs. (2) we re-express the above integral in the form

$$\tilde{A} = \frac{1}{(2\pi)^6} \int d\mathbf{R}' d\mathbf{R} d\mathbf{x}' d\mathbf{x} \times e^{-i(\mathbf{W}' \cdot \mathbf{R}' + \mathbf{b}' \cdot \mathbf{x}' - \mathbf{W} \cdot \mathbf{R} - \mathbf{b} \cdot \mathbf{x})} A_{\mathbf{R}'\mathbf{R}}(\mathbf{x}', \mathbf{x}) ,$$

where we denote $\mathbf{W} = \mathbf{k} + \mathbf{p}$, the momentum of the pair prior interaction and $\mathbf{b} = (\mathbf{k} - \mathbf{p})/2$, the relative momentum. Here again prime marks denote post interaction as shown in Fig. (2). To the above expression we apply the Wigner transform in the prior and post center of mass coordinates \mathbf{R} and \mathbf{R}' . Defining $\mathbf{Y} = \mathbf{R}' - \mathbf{R}$, and $\mathbf{Z} = (\mathbf{R}' + \mathbf{R})/2$ we obtain

$$\tilde{A} = \int \frac{d\mathbf{Z}}{(2\pi)^3} e^{i\mathbf{Z} \cdot (\mathbf{W}' - \mathbf{W})} g_{\mathbf{Z}}[\frac{1}{2}(\mathbf{W}' + \mathbf{W}); \mathbf{b}', \mathbf{b}] , \quad (\text{A1})$$

where $g_{\mathbf{Z}}$ represents a reduced interaction g at the coordinate \mathbf{Z} and is given by

$$g_{\mathbf{Z}}(\mathbf{K}_{||}; \mathbf{b}', \mathbf{b}) = \int \frac{d\mathbf{Y}}{(2\pi)^3} e^{-i\mathbf{Y} \cdot \mathbf{K}_{||}} \int d\mathbf{x}' d\mathbf{x} e^{-i(\mathbf{b}' \cdot \mathbf{x}' - \mathbf{b} \cdot \mathbf{x})} \times A_{\mathbf{Z} + \frac{1}{2}\mathbf{Y}, \mathbf{Z} - \frac{1}{2}\mathbf{Y}}(\mathbf{x}', \mathbf{x}) .$$

APPENDIX B: THE GROUND-STATE MIXED DENSITY

Let us denote the one-body density matrix in momentum space by $\tilde{\rho}(\mathbf{p}', \mathbf{p})$. In coordinate space this matrix is represented by $\rho(\mathbf{r}', \mathbf{r})$, where

$$\tilde{\rho}(\mathbf{p}', \mathbf{p}) = \frac{1}{(2\pi)^3} \int d\mathbf{r}' d\mathbf{r} e^{-i\mathbf{p}' \cdot \mathbf{r}'} \rho(\mathbf{r}', \mathbf{r}) e^{i\mathbf{p} \cdot \mathbf{r}} .$$

It is customary to introduce the center-of-mass $\mathbf{Z} = (\mathbf{r}' + \mathbf{r})/2$, and relative $\mathbf{x} = \mathbf{r}' - \mathbf{r}$, coordinates. Similarly, we define the mean $\mathbf{P} = (\mathbf{p}' + \mathbf{p})/2$, and transferred $\mathbf{Q} = \mathbf{p}' - \mathbf{p}$, momenta. With the use of these transformations we denote $\tilde{\rho}(\mathbf{p}', \mathbf{p}) \equiv \tilde{\rho}(\mathbf{Q}; \mathbf{P})$; $\rho(\mathbf{r}', \mathbf{r}) \equiv \rho(\mathbf{Z}; \mathbf{x})$. Therefore,

$$\begin{aligned} \tilde{\rho}(\mathbf{Q}; \mathbf{P}) &= \frac{1}{(2\pi)^3} \int d\mathbf{Z} d\mathbf{r} e^{-i(\mathbf{P} \cdot \mathbf{x} + \mathbf{Q} \cdot \mathbf{Z})} \rho(\mathbf{Z}; \mathbf{x}) ; \\ &\equiv \frac{1}{(2\pi)^3} \int d\mathbf{Z} e^{-i\mathbf{Q} \cdot \mathbf{Z}} \rho(\mathbf{Z}) G(\mathbf{Z}; \mathbf{P}) , \end{aligned} \quad (\text{B1})$$

where we have defined

$$\rho(\mathbf{Z}) G(\mathbf{Z}; \mathbf{P}) = \int d\mathbf{x} e^{-i\mathbf{P} \cdot \mathbf{x}} \rho(\mathbf{Z}; \mathbf{x}) .$$

In the Slater approximation for the mixed density its coordinate representation takes the form

$$\rho(\mathbf{Z}; \mathbf{x}) = \rho(\mathbf{Z}) F(Z; x) , \quad (\text{B2})$$

with $F(Z; x) = 3 j_1(\hat{k}_Z r)/(\hat{k}_Z r)$. Here \hat{k}_Z , the local Fermi momentum, depends on the local density $\rho(Z)$ through

$$\hat{k} = (3\pi^2 \rho)^{1/3} .$$

Within this approximation, and assuming spherical symmetry in the local density, the function $G(\mathbf{Z}; \mathbf{P})$ can be calculated directly. When used to evaluate Eq. (B1) for $\tilde{\rho}(\mathbf{Q}; \mathbf{P})$ we obtain

$$\tilde{\rho}(\mathbf{Q}; \mathbf{P}) = 4\pi \int_0^\infty Z^2 dZ j_0(QZ) \rho(Z) S_F(P; Z) , \quad (\text{B3})$$

where

$$S_F(P; Z) = \frac{1}{\frac{4}{3}\pi \hat{k}_Z^3} \Theta(\hat{k}_Z - P) . \quad (\text{B4})$$

APPENDIX C: EVALUATION OF $\int d\mathbf{Q} j_0 \Phi_1$

We evaluate the volume integral $\Omega_q(Z', Z)$ defined by

$$\Omega_q(Z', Z) \equiv \int d\mathbf{Q} j_0(QZ') \left[\frac{j_1(Z|\mathbf{Q} - \mathbf{q}|)}{Z|\mathbf{Q} - \mathbf{q}|} \right]$$

where we express $|\mathbf{Q} - \mathbf{q}| = \sqrt{Q^2 + q^2 - 2Qqu}$, with $u = \hat{Q} \cdot \hat{q}$. The solid-angle integration is straightforward, leading for Ω

$$\Omega_q(Z', Z) = \frac{4\pi}{Z^2 Z'} \int_0^\infty \frac{Q \sin QZ' dQ}{q^2 - Q^2} \times \left[\frac{\cos(QZ) \sin(qZ)}{qZ} - \frac{\sin(QZ) \cos(qZ)}{QZ} \right] .$$

This integral can be evaluated analytically and the expression depends on the location of Z relative to Z' . The two cases of interest are $Z > Z'$, and $Z < Z'$, where we obtain

$$\Omega_q(Z', Z) = \begin{cases} 0 & \text{if } Z < Z'; \\ \left(\frac{2\pi^2}{Z^3} \right) j_0(qZ') & \text{if } Z > Z'; \end{cases} \quad (\text{C1})$$

When $Z = Z'$ the result is finite but discontinuous. Due to its involved form we have preferred to omit it for clarity, keeping in mind that at $Z = Z'$ its contribution as integrand vanishes. Thus, without loss of generality we can express

$$\Omega_q(Z', Z) = \frac{2\pi^2}{Z^3} j_0(qZ') \times \Theta(Z - Z') , \quad (\text{C2})$$

with $\Theta(x)$ the Heaviside step function.

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- [22] By *unabridged* we mean the nine-dimensional integration comprised of six dimensions in momentum space ($d\mathbf{P}d\mathbf{Q}$) and three in coordinate space ($d\mathbf{Z}$). Spherical mass distribution reduces the dimension of non trivial integrals to seven (6+1).

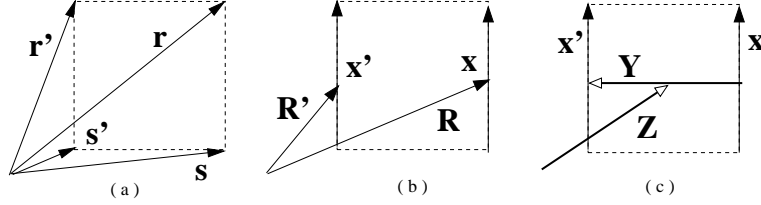


FIG. 1: Schematic representation of the vector coordinates in a two-body operator.

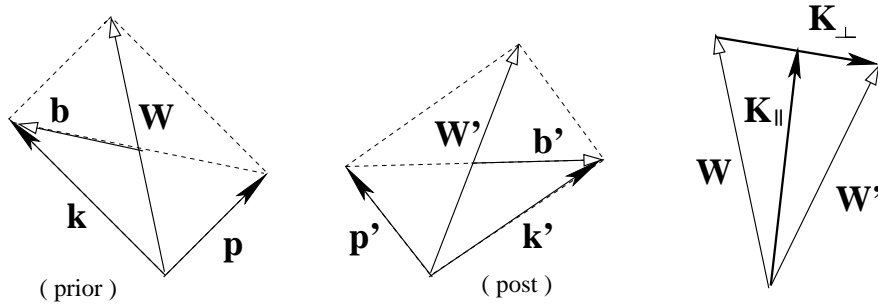


FIG. 2: Schematic representation of the momenta in a two-body operator.

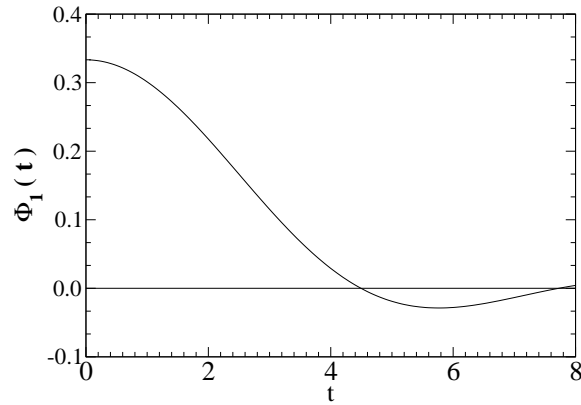


FIG. 3: The profile function $\Phi_1(t)$ as function of t .

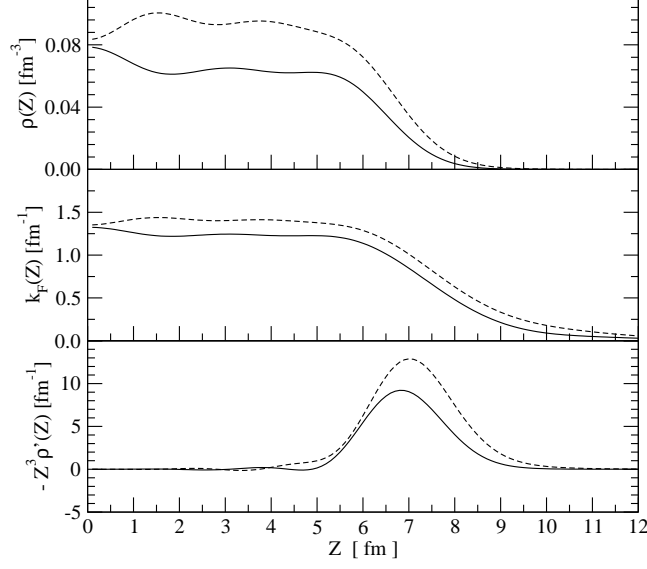


FIG. 4: Local density ρ (upper frame), local momentum \hat{k}_F (middle frame) and Z^3 times the negative gradient of the density (lower frame) as functions of the local coordinate Z . The solid and dashed curves represent different theoretical models.

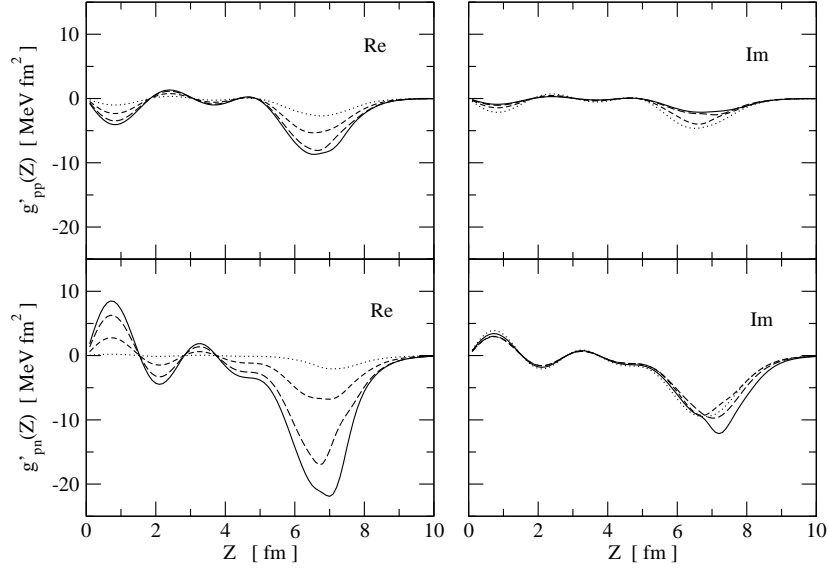


FIG. 5: Fermi integral of the gradient of the g matrix (Γ_{NN}) as function of the radial distance Z in ^{208}Pb . The pp (pn) channel is shown in the upper (lower) frame, whereas the real (imaginary) components are plotted in the left (right) panels. The solid, long-dashed, short-dashed and dotted curves represent results for Γ_{NN} at nucleon energies of 65, 100, 200 and 300 MeV, respectively.